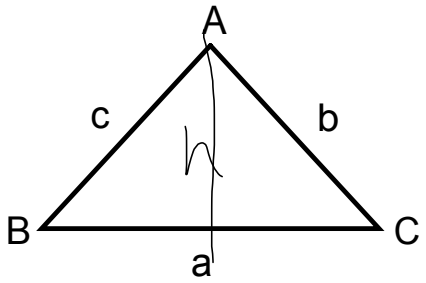
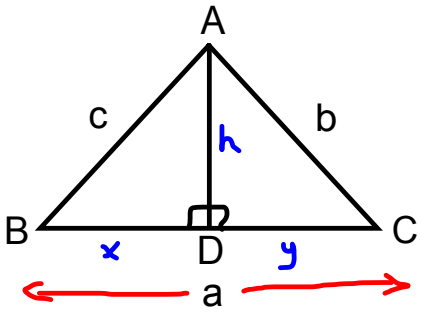
# The Cosine Law

## Developing the Law:

Given a non-right triangle ABC;




Draw "h" to be perpendicular to side a and meet at BC at D. Let x represent the length of BD and let y represent the length of DC.




In Triangle ABD,

$$c^2 = h^2 + x^2$$
$$c^2 - x^2 = h^2$$

In Triangle ACD,

$$b^2 = h^2 + y^2$$
$$b^2 - y^2 = h^2$$

*and*

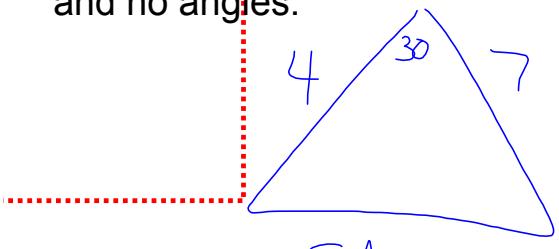
$$\cos C = \frac{y}{b}$$
$$y = b\cos C$$

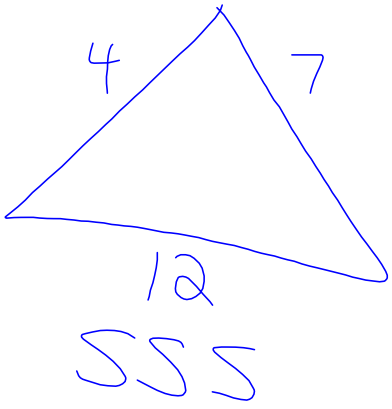
$$c^2 - x^2 = b^2 - y^2$$
$$c^2 = b^2 - y^2 + x^2$$
$$c^2 = b^2 - y^2 + (a-y)^2$$
$$c^2 = b^2 - y^2 + a^2 - 2ay + y^2$$
$$c^2 = a^2 + b^2 - 2ay$$
$$c^2 = a^2 + b^2 - 2a(b\cos C)$$

## Cosine Law

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

The cosine law must be used in a non-right triangle that has two sides with a contained (trapped) angle or when there is all three sides and no angles.



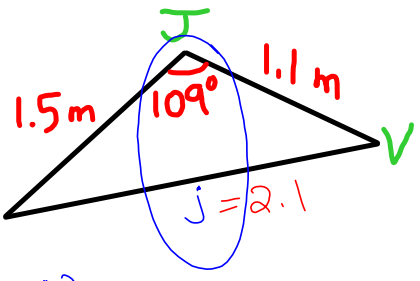



# Cosine Law Day # 2

Intro. Work:

Solve each triangle. Round all angle measures to the nearest degree and all side lengths to one decimal place.

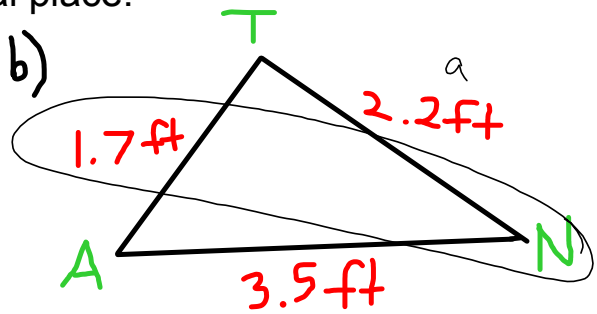
a)




$j^2 = v^2 + p^2 - 2vp\cos J$

$j^2 = 1.5^2 + 1.1^2 - 2(1.5)(1.1)\cos 109°$

b)




$\frac{a^2 + t^2 - n^2}{2at} = \cos N$

$\frac{2.2^2 + 3.5^2 - 1.7^2}{2(2.2)(3.5)} = \cos N$

Pull

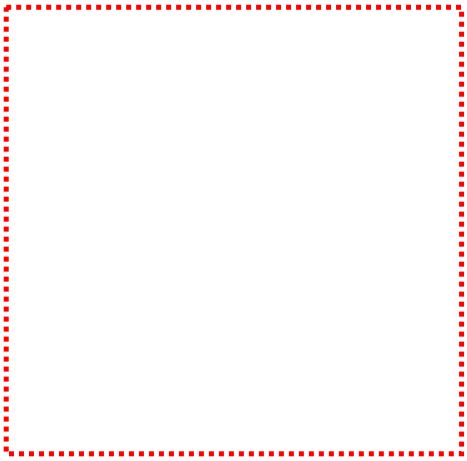
Pull

Example # 1: A triangle lot sits at the corner of two streets that intersect an an angle of 58°. One street side of the lot is 32 m and the other is 40 m. How long is the back of the lot (the third side), to the nearest meter?

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = (40)^2 + (32)^2 - 2(40)(32)\cos 58^\circ$$
$$a^2 = 1024 + 1600 - 2560\cos 58^\circ$$
$$a^2 = 2624 - 2560\cos 58^\circ$$
$$a^2 = 2624 - 1356.5933$$
$$a^2 = 1267.4067$$
$$a = \sqrt{1267.4067}$$
$$a = 35.6$$

Therefore

Example # 2: A bicycle race follows a triangular course. The three legs of the race are, in order, 2.3 km, 5.9 km and 6.2 km. Find the angle between the starting leg and the finishing leg to the nearest degree.



Homework p. 325 # 3 - 8 , 10